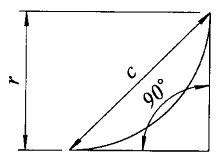
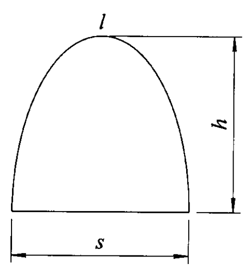
1306 GEOMETRIC SOLUTIONS

1306-1 Areas of Plane Figures



Spandrel

Area = $0.2146r^2$ = $0.1073c^2$
Example r = 3
Area = 0.2146 x 3^2= 1.0314 Ans.



Parabola

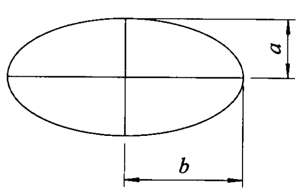
l = length of curvedline = periphery - s

$$l = \frac{s^2}{8h}\left[\sqrt{c(1+c)} + 2.0326 \text{ x } \log\left(\sqrt{c} + \sqrt{1+c}\right)\right] \text{ in which } c = \left(\frac{4h}{s}\right)^2$$

Area = $\frac{l}{3} sh$

Example s = 3; h = 4;

Area = $\frac{l}{3}$ x3x4=8 Ans.

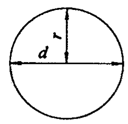


Ellipse

Area = π ab = 3.1416 ab

Circum = $2\pi\sqrt{\frac{a^2+b^2}{2}}$
(close approximation)

Example a = 3; b =4
Area = 3.1416 x 3 x 4 = 37.6992 Ans.
Circum = 2 x 3.1416 x $\sqrt{\frac{3^2+4^2}{2}}$ = 6.2832 x $\sqrt{12.5}$ = 6.2832 x 3.5355 = 22.21 Ans.



Circle

π = 3.1416; A = area; d = diameter; p = circumference or periphery;
r = radius;

$p = \pi d = 3.1416d$. $\quad p = 2\sqrt{\pi A} = 3.54\sqrt{A}$

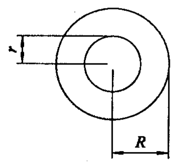
$p = 2\pi r = 6.2832\,r$. $\quad p = \frac{2A}{r} = \frac{4A}{d}$

$d = \frac{p}{\pi} = \frac{p}{3.1416}$ $\quad d = 2\sqrt{\frac{A}{\pi}} = 1.128\sqrt{A}$

$r = \frac{p}{2\pi} = \frac{p}{6.2832}$ $\quad r = \sqrt{\frac{A}{\pi}} = 0.564\sqrt{A}$

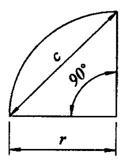
$A = \frac{rd^2}{4} = 0.7854\,d^2$ $\quad A = \frac{p^2}{t\pi} = \frac{p^2}{12.57}$

$A = \pi r^2 = 3.1416\,r^2$ $\quad A = \frac{pr}{2} = \frac{pd}{4}$



Circular Ring

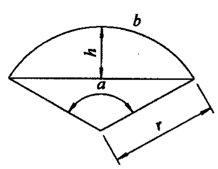
Area = $\pi\,(R^2 - r^2) = 3.1416\,(R^2 - r^2)$
Area = $0.7854\,(D^2 - d^2) = 0.7854\,(D-d)(D+d)$
Area = difference in areas between the inner and outer circles.
Example. $R = 4$; $r = 2$
Area = $3.1416(4^2 - 2^2) = 37.6992$ Ans.



Quadrant

Area = $\frac{\pi r^2}{4} = 0.7854\,r^2 = 0.3927\,c^2$

Example. $r = 3$; c = chord
Area = $.7854 \times 3^2 = 7.0686$ Ans.



Segment

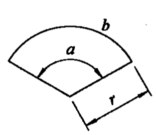
b = length of arc, a = angle in degrees
c = chord = $\sqrt{4\,(2hr-h^2)}$
Area = 1/2 $[br-c(r-h)]$

or $\quad = \pi r^2 \frac{a}{360} - \frac{c(r-h)}{2}$

When o is greater than 180° than $\frac{c}{2}$ x difference between r and h is added to the fraction $\frac{\pi r^2}{360}$

Example. $r = 3$; $a = 120°$; $h = 1.5$

Area = $3.1416 \times 3^2 \times \frac{120}{360} - \frac{5.196(3-1.5)}{2} = 5.5278$ Ans.

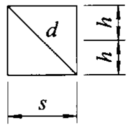


Sector

Area = $\frac{br^2}{2}$ or $\pi r^2 \frac{a}{360°}$

a = angle in degrees; b = length of arc
Example. $r = 3$; $a = 120°$

Area = $3.1416 \times 3^2 \times \frac{120}{360} = 9.4248$ Ans.



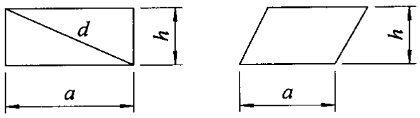
Square

Diagonal = $d = s\sqrt{2}$

Area = $s^2 = 4b^2 = 0.5d^2$

Example. $s = 6{:}\ b = 3.\ Area = (6)^2 = 36\ Ans.$

$d = 6 \times 1.414 = 8.484\ Ans$

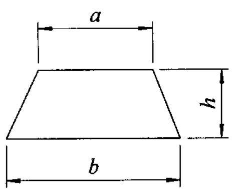


Rectangle and Parallelogram

Area = ab or $b\sqrt{d^2 - b^2}$

Example $a = 6{:}\ b = 3.$

Area = $3x6 = 18\ Ans.$

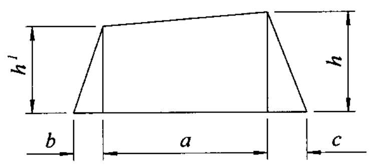


Trapezoid

Area = $1/2\ h(a + b)$

Example. $a = 2;\ b = 4;\ h = 3.$

Area = $1/2 \times 3(2+4) = 9\ Ans.$

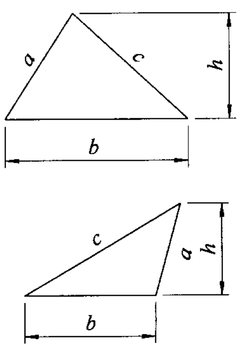


Trapesium

Area = $1/2[a(h+h^1) + bh^1 + ch]$

Example. $a = 4;\ b = 2;\ c = 2;\ h = 3;\ h^1 = 2.$

Area = $1/2[4(3+2)+(2x2)+(2x3)] = 15\ Ans$



Triangles

Both formulas apply to both figures

Area = $1/2bh$

Example. $h=3;\ b=5$

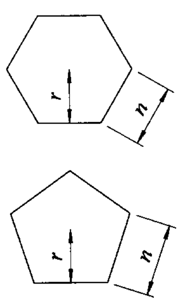
Area = $1/2(3x5)=7\ 1/2\ Ans.$

Area = $\sqrt{S(S-a)(S-b)(S-c)}$ when $S = \dfrac{a+b+c}{2}$

Example. $a=2;\ b=3;\ c=4$

$$S = \frac{2+3+4}{2} = 4.5$$

$$\text{Area} = \sqrt{4.5(4.5-2)(4.5-3)(4.5-4)} = 2.9\ Ans$$



Regular Polygons

Area	5 sides = 1.720477 S^2	= 3.63271 r^2
	6 sides = 2.598150 S^2	= 3.46410 r^2
	7 sides = 3.633875 S^2	= 3.37101 r^2
	8 sides = 4.828427 S^2	= 3.31368 r^2
	9 sides = 6.181875 S^2	= 3.27573 r^2
	10 sides = 7.894250 S^2	= 3.24920 r^2
	11 sides = 9.365675 S^2	= 3.22993 r^2
	12 sides = 11.196300 S^2	= 3.21539 r^2

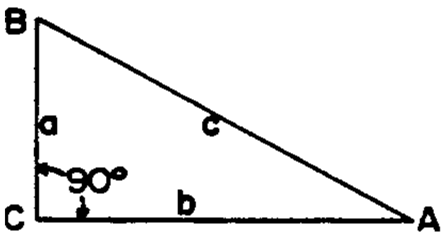
n= number of sides r= short radius

S= length of side R= long radius

$$\text{Area} = \frac{n}{4} S^2 \cot. \frac{180^\circ}{n} = \frac{n}{2} R^2 \sin. \frac{360^\circ}{n} = nr^2 \tan. \frac{180^\circ}{n}$$

1306-2 Triangles

RIGHT TRIANGLE



(1) sin A = a / c = cos B

(2) cos A = b / c = sin B

(3) tan A = a / b = cot B

(4) cot A = b / a = tan B

(5) sec A = c / b = csc B

(6) csc A = c / a = sec B

(7) vers A = 1 - cos A = 1 - b /c

(8) exsec A = sec A - 1 = c/b -1

(9) a = $\sqrt{(c+b)(c-b)}$

(10) b = $\sqrt{(c+a)(c-a)}$

(11) c = $\sqrt{(a)^2 + (b)^2}$

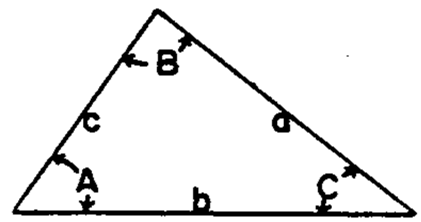
(12) Area = (1/2) a b

(13) Area = (1/2) b^2 tan A

Trigonometric Functions of any Angle

$\sin (90^0 + \theta) = \cos \theta$
$\cos (90^0 + \theta) = - \sin \theta$
$\tan (90^0 + \theta) = - \cot \theta$
$\cot (90^0 + \theta) = - \tan \theta$

OBLIQUE TRIANGLE



(1) Law of Sines
(When two angles and included side are known)

(sin A) /a = (sin B) / b = (sin C)/c

(2) Law of Tangents
(When two sides and the included angle are known)

(a +b) / (a -b) =
(tan (1/2) (A + B)) / (tan (1/2) (A-B))

(3) Law of Cosines
(When two sides and the included angle are known or when all three sides are known)

$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos A$

(4) Half-angle formula
(when all three sides are known)*

* s = (1 / 2) (a + b + c)

$\sin (1/2)A = \sqrt{\frac{(s-b)(s-c)}{bc}}$

(5) Area = (1/2)ab sin C
Area = (1/2)bc sin A
Area = (1/2)ac sin B